MAURER Vibration Isolation

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Vibration Isolation

Vibrations due to kinematic and force excitations

Undesirable and inacceptable vibrations in buildings and rooms with sensitive equipment such as microscopes, lasers and other vibration sensitive devices are caused by the following two ways of excitations (Fig. 1, left):

- **Kinematic excitation**: ground vibrations with displacement $\mathbf{x_{ex}(t)}$ due to traffic, earthquake, wind, explosions and other loadings excite the sensitive equipment.
- **Force excitation**: inertial forces $\mathbf{f_{ex}(t)}$ of rotating machines excite floors and thereby entire buildings.

Assuming very stiff bearings or even the absence of bearings between ground and sensitive equipment and between rotating machine and floor/building, respectively, the disturbing vibrations and forces, respectively, are transmitted to the sensitive equipment and floor/building almost without any attenuation. The resulting vibrations $\mathbf{x(t)=x_{ex}(t)}$ and forces $\mathbf{f(t)=f_{ex}(t)}$ are not acceptable and may lead to premature material fatigue.

Different kinds of excitations exist (Fig. 2). Rotating machines often do not generate harmonic but periodic forces. Punching machines lead to impulse-type force excitation. Wind (no vortex shedding) and earthquakes lead to broad band excitations.

Vibration isolation system

The vibration isolation targets to **minimize the transmissibility** of displacement in case of ground excitation by so-called "passive" isolation and the transmissibility of force in case of machine induced vibrations by "active" isolation (Fig. 1, right). Both isolation systems are identical and consist of a spring packet in parallel to a dash pot damper.

Single degree-of-freedom system

Equation of motion

The dynamics of the isolated structure are conveniently analysed by the model of the single degree-of-freedom system (Fig. 1, right). Its equations of motion for force (1) and kinematic (2) excitations become

$$m\ddot{x}+c\dot{x}+k_{dyn}\,x=f_{ex} \tag{1}$$

$$m\ddot{x}+c\left(\dot{x}-\dot{x}_{ex}\right)+k_{dyn}\left(x-x_{ex}\right)=0 \tag{2}$$

with:

- $\mathbf{m}$: mass of system to be isolated in case of ground excitation and the mass of machine (without accelerated machine parts) in case of force excitation,
- $\mathbf{c}$: viscous damper coefficient of oil damper,
- $\mathbf{k_{dyn}}$: dynamic spring stiffness of spring packet.

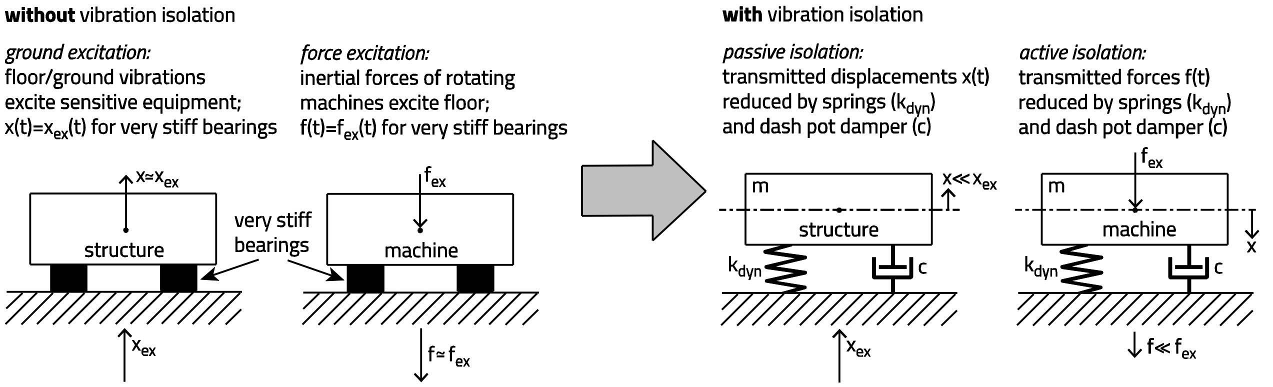

Fig. 1 – Vibrations in sensitive equipment and structures without (left) and with (right) vibration isolation

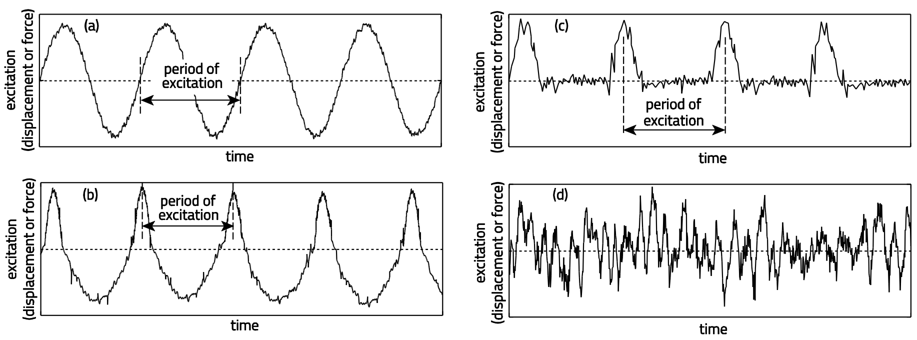

Fig. 2 – Harmonic (a), periodic (b), impuls-type (c) and broad band (d) excitations

Natural frequency

The natural, i.e. undamped frequency in Hertz of the single degree-of-freedom oscillator is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_{dyn}}{m}} \qquad (3)$$

where the dynamic stiffness k_{dyn} describes the stiffness of the spring element under dynamic loading. For spiral steel springs k_{dyn} is identical to the static stiffness k while k_{dyn} represents a linearized value for elastomer springs. In practice k_{dyn} is usually given in N/mm which leads to the following approximation

$$f_0\,[Hz] = \frac{\sqrt{1000}}{2\pi}\sqrt{\frac{k_{dyn}\,[N/mm]}{m[kg]}} \approx 5\sqrt{\frac{k_{dyn}\,[N/mm]}{m[kg]}} \qquad (4)$$

Static spring deflection

The static deflection of the spring due to the load of mass m is (g : gravitational acceleration)

$$\Delta h = \frac{gm}{k} = \frac{g}{(2\pi f_0)^2} \qquad (5)$$

which shows that Δh can be expressed as function of the natural frequency only (Fig. 3). For Δh in mm and f_0 in Hertz the following approximation is commonly used

$$\Delta h\,[mm] \approx \frac{250}{f_0^2\,[Hz]} \qquad (6)$$

Damping ratio

The damping ratio ζ of the oscillator is given by the viscous damper coefficient c of the oil damper

$$\zeta = \frac{c}{2\sqrt{k_{dyn}\,m}} = \frac{c}{2m(2\pi f_0)} = \frac{c(2\pi f_0)}{2k_{dyn}} \qquad (7)$$

The expression (7) assumes linear viscous damping of the passive oil damper which is fulfilled for silicon oil based dash pot dampers.

Free decay response

The free decay response of the damped single degree-of-freedom oscillator is characterized by a harmonic oscillation at damped frequency f_d and an exponential function describing the decay of the peaks X (Fig. 4)

$$x(t) = x_{t=0}\, e^{-(\zeta\, 2\pi f_0\, t)} \cos(2\pi f_d\, t) \qquad (8)$$

$$f_d = f_0\sqrt{1-\zeta^2} = 1/T_d \qquad (9)$$

For oscillators with linear damping the ratio of subsequent peaks during the free decay response yields the logarithmic decrement

$$\delta = \ln\left(\frac{X_n}{X_{n+1}}\right) \qquad (10)$$

from which the damping ratio can be derived as follows

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \qquad (11)$$

The approximation $\zeta \approx \delta/2/\pi$ is valid for small ζ (<10%).

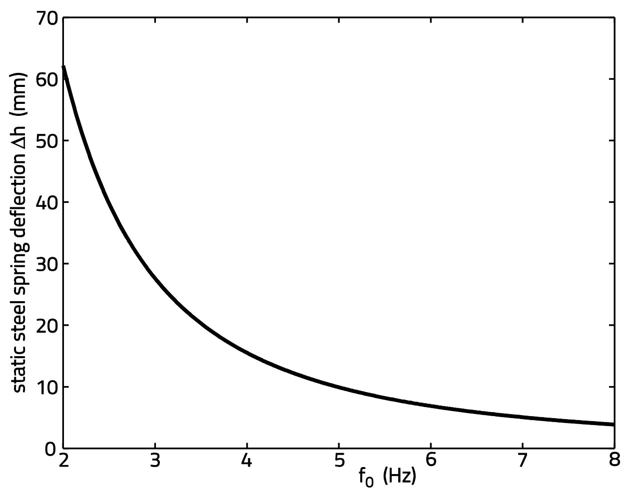

Fig. 3 – Relation between static steel spring deflection and natural frequency of isolation system

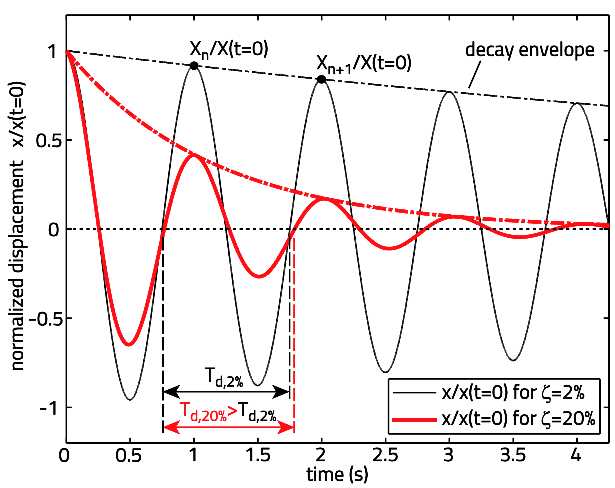

Fig. 4 – Free decay response of a single degree-of-freedom oscillator with linear damping

Forced excitation

If the single degree-of-freedom oscillator is excited by the base displacement or the machine-induced force the **single degree-of-freedom system vibrates at the frequency of excitation** f_{ex}. If the excitation frequency equals the natural frequency, i.e. $f_{ex} = f_0$, **resonant** vibration with extremely large amplitudes occur whose magnitude depends on the damping ratio only.

Amplification function

The displacement amplification of a single degree-of-freedom system with force excitation is expressed by the displacement amplitude divided by the static deflection of the single degree-of-freedom system due to the excitation force amplitude (Fig. 5)

$$\left|\frac{x}{f_{ex}/k_{dyn}}\right| = \frac{X}{X_{static}} = \frac{1}{\sqrt{\left(1-\lambda^2\right)^2 + \left(2\zeta\lambda\right)^2}} \tag{12}$$

where λ denotes the frequency ratio

$$\lambda = \frac{f_{ex}}{f_0} \tag{13}$$

Transfer function

The impact of the machine forces on the forces acting on the structure is expressed by the absolute **transfer function** $|\alpha|$ **between transmitted force and excitation force** (Fig. 6)

$$|\alpha| = \left|\frac{f}{f_{ex}}\right| = \frac{F}{F_{ex}} \tag{14}$$

and becomes for harmonic excitation and **very small flexibility**, i.e. for infinitely high impedance of the floor

$$|\alpha| = \sqrt{\frac{1+\left(2\zeta\lambda\right)^2}{\left(1-\lambda^2\right)^2 + \left(2\zeta\lambda\right)^2}} \tag{15}$$

The transfer function for kinematic excitation, i.e. base excitation, for small base vibration amplitudes takes the same form as (15) (Fig. 6)

$$|\alpha| = \left|\frac{x}{x_{ex}}\right| = \frac{X}{X_{ex}} = \sqrt{\frac{1+\left(2\zeta\lambda\right)^2}{\left(1-\lambda^2\right)^2 + \left(2\zeta\lambda\right)^2}} \tag{16}$$

For small damping ratios ($\zeta < 5\%$) the transfer function $|\alpha|$ only depends on the frequency ratio λ

$$|\alpha|_{\zeta<5\%} \approx \frac{1}{\lambda^2 - 1} \quad (\zeta < 5\%) \tag{17}$$

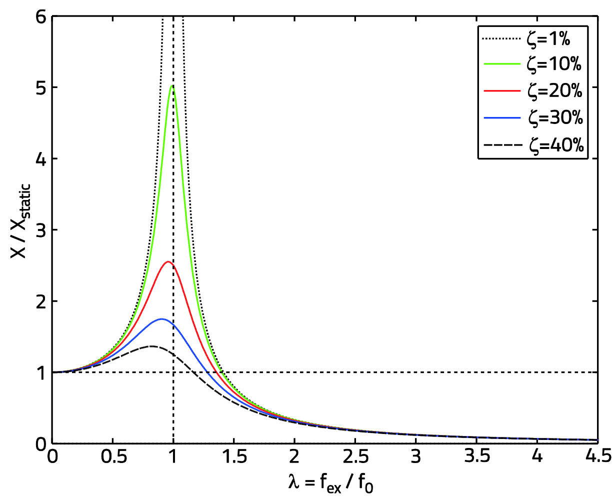

Fig. 5 – Amplification function for different damping ratios ζ of single degree-of-freedom system

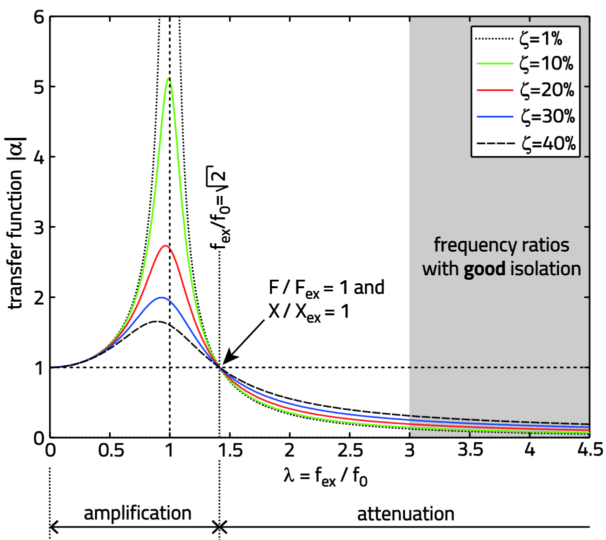

Fig. 6 – Transfer function $|\alpha|$ for different damping ratios ζ of isolation system

Transmissibility

The **force transmissibility** for force excitation and the **displacement transmissibility** for base excitation are defined by $|\alpha|$ in decibel (dB)

$$|\alpha|_{dB} = 20\log_{10}(|\alpha|) \qquad (18)$$

Some typical values of $|\alpha|$ and $|\alpha|_{dB}$ are given in Table 1.

Table 1 – Typical values of $|\alpha|$ and $|\alpha|_{dB}$

$\|\alpha\|$	0.01	0.1	0.5	1	2	10	100
$\|\alpha\|_{dB}$	-40	-20	-6	0	6	20	40

The transmissibility function $|\alpha|_{dB}$ (Fig. 7) for different damping ratios ζ of the isolation system shows the following characteristics:

- **Amplification,** $f_{ex} < \sqrt{2}\, f_0$: Transmitted amplitudes are larger than those of excitation, are larger than without isolation system and can only be limited by the damping of the isolation system.
- **Attenuation,** $f_{ex} > \sqrt{2}\, f_0$: Transmitted amplitudes are smaller than those of excitation. Notice that the isolation system reduces the amplitudes but cannot cancel the vibrations of the isolated structure.
- **Same phase and amplitude,** $f_{ex} < (0.2-0.3)\, f_0$: The transmitted amplitudes are approx. equal to and approx. in phase with those of excitation.
- **Good isolation,** $\lambda = f_{ex}/f_0 \geq 3$: Good isolation is achieved if the isolation frequency f_0 is **at least 3 times** lower than the lowest excitation frequency f_{ex} .

Trade-off behaviour of damping

High values of ζ **reduce resonant** amplitudes but **lower the attenuation** in the frequency range $\lambda > \sqrt{2}$, i.e. **lower a good roll-off behaviour**; the opposite effect is observed for small values of ζ . Thus, the optimal design of ζ strongly depends on the specifications of each project.

Displacement amplification

Due to the isolation system vibration amplitudes of elastically supported machine with inertial excitation are amplified. This is expressed by the amplification function

$$|\beta| = \frac{X}{\left(\dfrac{F_{ex}}{m(2\pi f_{ex})^2}\right)} = \frac{\lambda^2}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \qquad (19)$$

which is defined as the ratio between the displacement amplitude of the machine and the displacement of the machine without isolation system due to the excitation force.

The maximum steady state displacements of the machine occur at resonant excitation ($\lambda = 1$, Fig. 8). Therefore, the rotational speed of the machine should be changed fast during starting and stopping processes when the rotational frequency is in the vicinity of f_0 as transient amplitudes are smaller than their steady state values.

In the region of good isolation $(\lambda \geq 3, |\beta| \approx 1)$ the displacement amplitude of the machine does hardly depend on ζ but is in proportion to the excitation force amplitude. Within this frequency range ($\lambda \geq 3$) and for small damping ratio ($\zeta < 5\%$) $|\beta|$ may be approximated as follows

$$|\beta|_{\zeta<5\%} \approx \frac{\lambda^2}{\lambda^2 - 1} \qquad (\zeta < 5\%) \qquad (20)$$

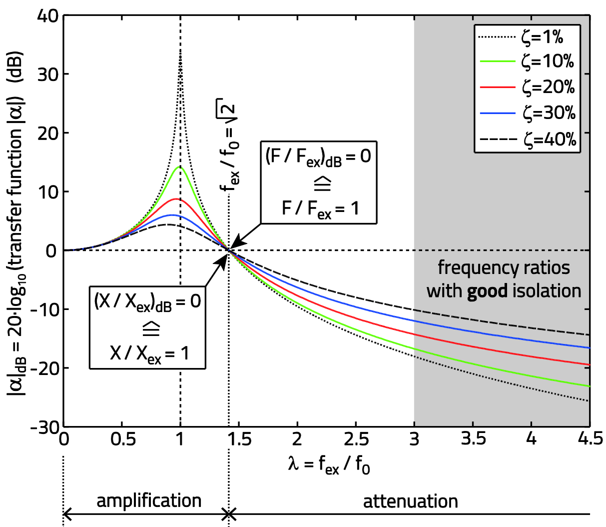

Fig. 7 – Transmissibility $|\alpha|_{dB}$ for different damping ratios ζ of isolation system

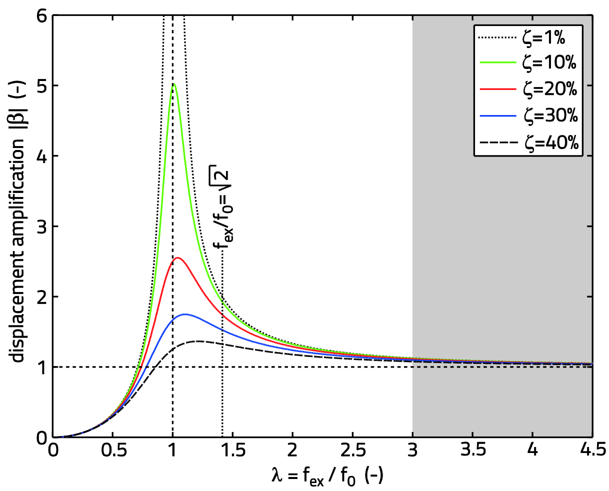

Fig. 8 – Displacement of vibrating machine for different damping ratios ζ of isolation system

Isolation efficiency

The isolation efficiency i is defined in % for the region of attenuation ($\lambda > \sqrt{2}$) only (Fig. 9)

$$i\,[\%] = \left(1 - |\alpha|\right) \cdot 100 \qquad (21)$$

which can be approximated for small damping ($\zeta < 5\%$)

$$i_{\zeta<5\%}\,[\%] \approx \left(\frac{\lambda^2 - 2}{\lambda^2 - 1}\right) \cdot 100 \quad (\zeta < 5\%) \qquad (22)$$

Expression (22) can be solved for the frequency ratio λ and thereby for the natural frequency f_0

$$f_0 \approx f_{ex} \sqrt{\frac{100 - i}{200 - i}} \quad (\zeta < 5\%) \qquad (23)$$

Putting the **required isolation efficiency** i_{req} into (23) yields the **required natural frequency**

$$f_{0,req} \approx f_{ex} \sqrt{\frac{100 - i_{req}}{200 - i_{req}}} \quad (\zeta < 5\%) \qquad (24)$$

The closed-form solution (24) may be used to calculate $f_{0,req}$ that ensures i_{req} whose recommended value is

$$i_{req} \approx 80\%...90\% \qquad (25)$$

Assuming i_{req} =80% in (24) leads to $f_{0,req} \approx f_{ex} / 2.45$. Even fairly small damping of 20%, which may not be sufficient for the mitigation of resonant vibrations, $f_{0,req} \approx f_{ex} / 2.45$ and ζ =20% (point a in Fig. 9) lead to $i \approx$ 73% whereby i_{req} =80% is not fulfilled. This example clearly demonstrates that the optimum design of f_0 and ζ requires computing the general formulation (21) of the isolation efficiency. Assuming ζ =30%, which generates sufficient mitigation of resonant vibrations, leads to $f_0 \approx f_{ex} / 3.5$ and ensures $i \geq i_{req}$ (point b in Fig. 9). Thus, the **isolation efficiency allows deriving the optimum design of the isolation system according to Client's specifications**.

Design by MAURER

MAURER offers the optimal design of the vibration isolation system according to Client's specifications. The optimal design consists of the following steps:

1. Client's information:
   - required isolation efficiency i_{req}
   - lowest excitation frequency f_{ex}^{lowest}
   - vertical loads
   - design for **traffic** induced base excitation only or also for **earthquake** induced vibrations
2. Select the lowest frequency ratio $\lambda^{lowest} = f_{ex}^{lowest} / f_0$ based on the recommendation λ^{lowest} =2.5...3.
3. Compute f_0 based on λ^{lowest} and f_{ex}^{lowest}.
4. Compute i for λ^{lowest} and various damping ratios ζ and select this damping ratio that fulfils $i \geq i_{req}$.

Steps 2 to 4 require several iterations to find the best solution that guarantees $i \geq i_{req}$ and sufficient mitigation of resonant amplitudes at $\lambda = 1$.

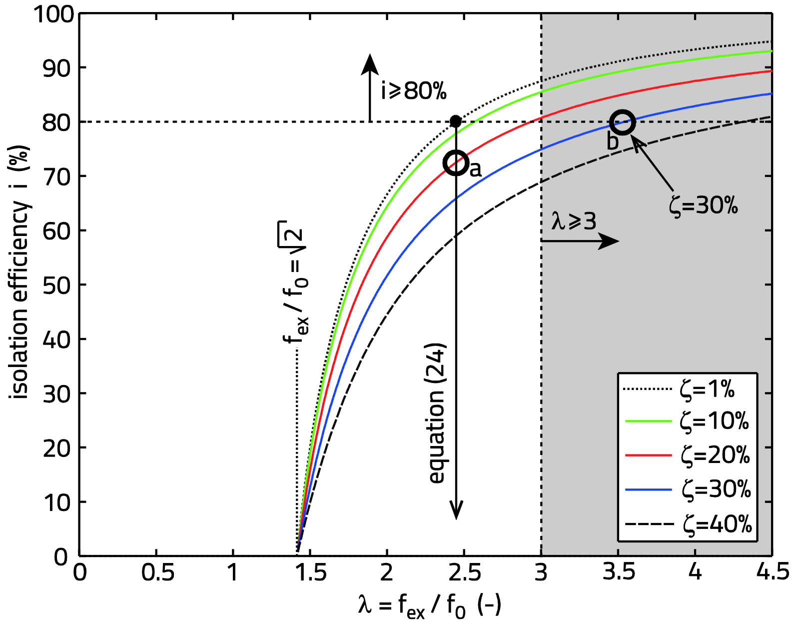

Fig. 9 – Isolation efficiency for different damping ratios ζ of isolation system

Transfer function for flexible structures

The impedance of the structure Z_2 must be considered for the computation of the transfer function $|\alpha|$ if (Fig. 10, left):

- the **flexibility of the floor structure** cannot be neglected, i.e. Z_2 cannot be assumed to be infinitely high, and
- the modal mass m_2 of the floor structure is **less than 10 times** the mass m_1 of the machine.

For the derivation of the transfer function $|\alpha|$ in its general format the model of the single degree-of-freedom system (Fig. 1, right) must be replaced by a two degree-of-freedom system describing the dynamics of the elastically supported machine mass and of the flexible structure (Fig. 10, right). The impedance of the machine

$$Z_1 = j\left(2\pi f_{ex}\right) m_1 \qquad (26)$$

(j : imaginary unit) and the impedance of the floor structure

$$Z_2 = j\left(2\pi f_{ex}\right) m_2 + c_2 + \frac{k_2}{j\left(2\pi f_{ex}\right)} \qquad (27)$$

yield the complex tansfer function α

$$\alpha = \frac{1+2\,j\,\zeta_1\,\lambda_1}{1-\dfrac{\lambda_1^2}{1+\left(\bar{Z}_1/\bar{Z}_2\right)}+2\,j\,\zeta_1\,\lambda_1} \qquad (28)$$

which eventually gives the absolute transfer function

$$|\alpha| = \frac{F}{F_{ex}} = \sqrt{\frac{1+\left(2\,\zeta_1\,\lambda_1\right)^2}{\left(1-\dfrac{\lambda_1^2 A}{A^2+B^2}\right)^2+\left(2\,\zeta_1\,\lambda_1+\dfrac{\lambda_1^2 B}{A^2+B^2}\right)^2}} \qquad (29)$$

with

$$A = 1+\frac{\frac{m_1}{m_2}\lambda_2^2\left(\lambda_2^2-1\right)}{\left(1-\lambda_2^2\right)+\left(2\,\zeta_2\,\lambda_2\right)^2}\,,\quad B = \frac{2\,\frac{m_1}{m_2}\,\zeta_2\,\lambda_2^3}{\left(1-\lambda_2^2\right)+\left(2\,\zeta_2\,\lambda_2\right)^2} \qquad (30)$$

where $\lambda_1 = f_{ex}/f_{0,1}$ and $\lambda_2 = f_{ex}/f_{0,2}$.

Vibration mitigation of piping systems

Piping systems in power stations may be excited by machines through the floor structure and the supports of the piping system (Fig. 11). Due to the high flexibility and low damping of pipes the resulting vibration amplitudes may be inacceptable large. **Viscous dampers are installed between pipe and additional damper supports to dissipate energy whereby the piping system damping is augmented** and consequently **oscillation amlitudes are reduced**. The following issues must be considered:

- **Damper supports are required** in order to attach the viscous dampers at the **position of maximum kinetic energy** of vibration, i.e. at **anti-node position** of the relevant mode, for maximum vibration mitigation.
- **High damper support stiffness** is required for high damping efficiency.
- The **viscous damper coefficient** c **must be optimized to the frequency** of the predominantly vibrating mode.
- The selection of the silicon oil must take into account **operating temperature** of the damper.
- Higher vibration modes require the installation of several smaller viscous dampers.

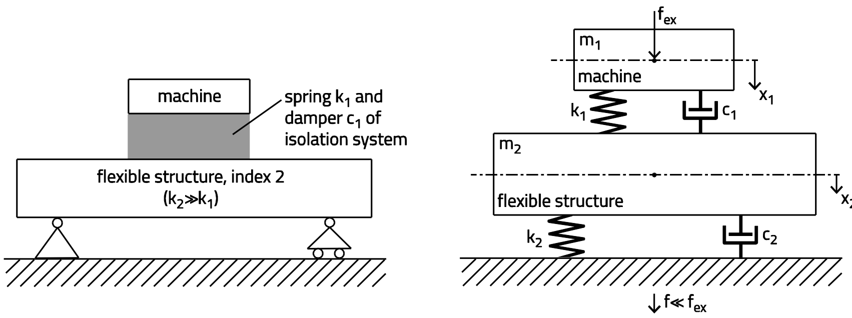

Fig. 10 – Isolation of machine induced vibrations on flexible structures

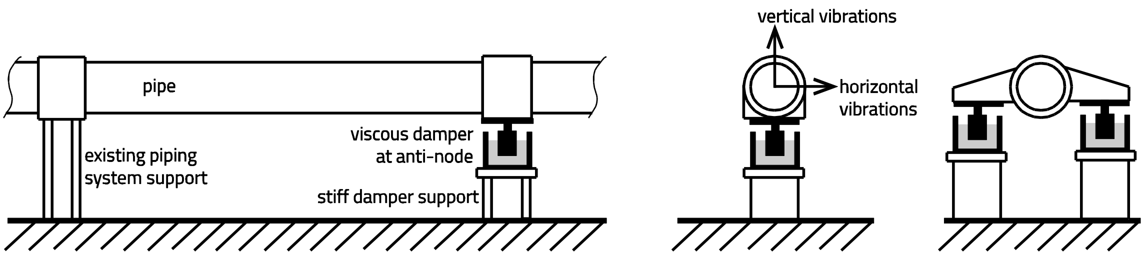

Fig. 11 – Reduction of spatial pipe vibrations by transverse viscous dampers

Vibration Isolation in Seismic Zones

Building vibrations due to earthquake excitation

For the vibration isolation of sensitive equipment / sensitive structures in buildings that are located in seismic zones vibration isolation becomes a demanding task because of the following facts (Fig. 12):

- **Mainly horizontal vibrations:** earthquakes excite buildings predominantly in horizontal direction.
- **Large horizontal displacements:** compared to typical vertical vibration amplitudes of isolation systems on the order of micro-meters the relative motion in the curved surface sliders in horizontal direction can be up to 0.5 m.
- **Low frequency excitation:** most earthquakes show their highest energy content within 0.5 Hz to 2 Hz which is below the frequency range of most isolation systems.

Solution by MAURER

MAURER as a leading specialist for earthquake protection systems provides **taylor-made solutions** to optimally combine the vibration isolation system with the technology of earthquake protection. Solutions include (Fig. 12):

- **Optimal design of earthquake protection and vibration isolation systems by dynamic non-linear simulation for accelerograms being equivalent to the specified elastic response spectrum.**
- Curved surface sliders embedded within elastomer pads.

Curved surface sliders allow for horizontal movement of the building relative to the shaking ground and thereby decouple the structure (step a in Figs. 13, 14). The energy dissipation by friction on the sliding surfaces augments the damping of the building which additionally reduces the acceleration response of the building (step b in Figs. 13, 14).

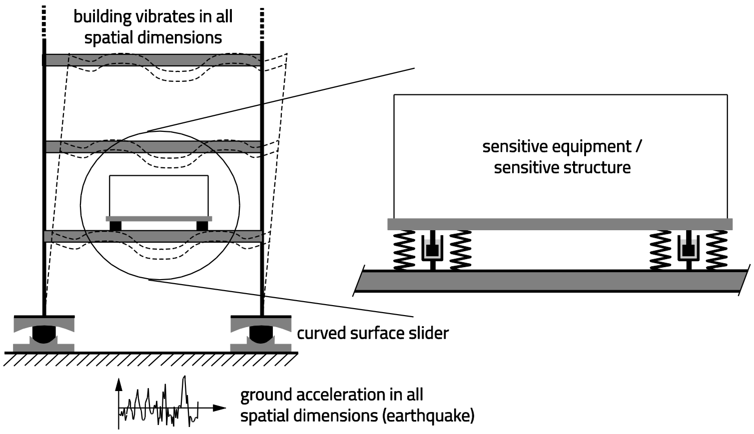

Fig. 12 – Possible solution for vibration isolation in seismic zones

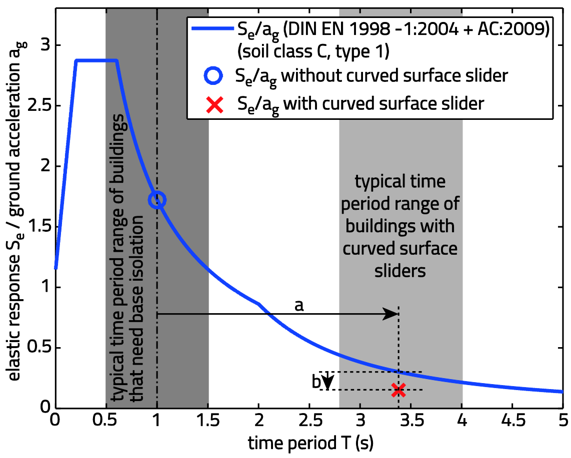

Fig. 13 – Reduction of horizontal building accelerations by curved surface sliders in the elastic response spectrum as function of time period

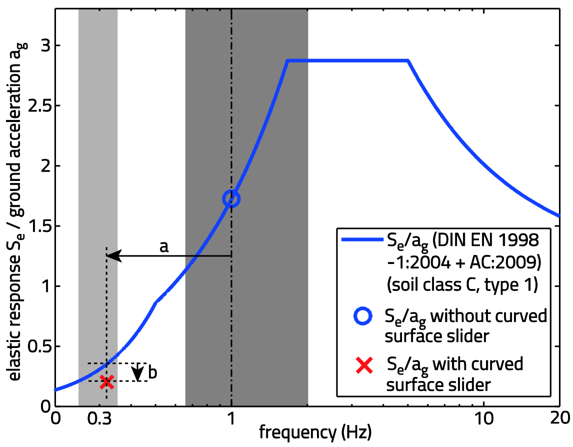

Fig. 14 – Reduction of horizontal building accelerations by curved surface sliders in the elastic response spectrum as function of frequency

Design Considerations

Fundament plate

An **additional stiff fundament plate** between machine and isolation system must be included if the steel frame of the machine is not stiff enough to ensure that all **spring elements** of the isolation system are **working in phase**. The mass of the additional fundament plate also helps to reduce the relative diplacement amplification $|\beta|$ of the machine because the mass m of the single degree-of-freedom system is augmented by the mass of the fundament plate whereby the term $F_{ex}/(m(2\pi f_{ex})^2)$ in (19) is reduced.

Force and moment equilibria

Typically, the machine with fundament plate is elastically supported by **an even number** of spring packets. To guarantee vertical operation of the isolation system without any bending the vertical load on all spring packets must be equal

$$\sum_i (F_i) = mg \qquad (31)$$

and the positions of the spring packets must ensure zero resulting moment in both horizontal directions x and y

$$\sum_i (F_i x_i) = 0, \sum_i (F_i y_i) = 0 \qquad (32)$$

For the example depicted in Fig. 15 with elastic supports in the four corners (1, 1', 3, 3') and given centre of mass x_g equation (32) leads to

$$x_2 = 3x_g - x_3 \qquad (33)$$

Steel spiral springs

Basically three types of spring elements exist:

- Elastomer pads that generate coupled non-linear stiffness and approx. linear damping behaviours; used for rather **high isolation frequencies** $f_0 > 6$ Hz.
- **steel spiral springs that are characterized by linear stiffness** behaviour, i.e. $k_{dyn} = k$, **very small (linear) damping** on the order of 0.3% to 0.5%; adopted for **medium isolation frequencies 2 Hz** $< f_0 <$ **10 Hz**.
- air springs that are used when lowest isolation frequencies are required ($f_0 > 0.5$ Hz).

Vibration isolation systems based on steel spiral springs generate optimum isolation due to the following benefits of steel sprial springs:

- **linear stiffness** behaviour whereby the isolation frequency does not depend on the relative motion amplitude of the isolation system,
- **very small and linear damping** ratio whereby the required damping ratio of the vibration isolation system can be generated by an optimally tuned linear dash pot damper,
- **accurate design** is possible, and
- plastic deformation hardly occurs.

Linear viscous damper

Except for elastomeric bearings the required damping of the isolation system must be produced by a dash pot damper. The following issues must be addressed:

- the dash pot damper must be designed to dissipate energy in all three spatial dimensions,
- the actual viscous damper coefficient must be equal to its design value c (7) at operating temperature of the damper, and
- the damper support must be stiff.

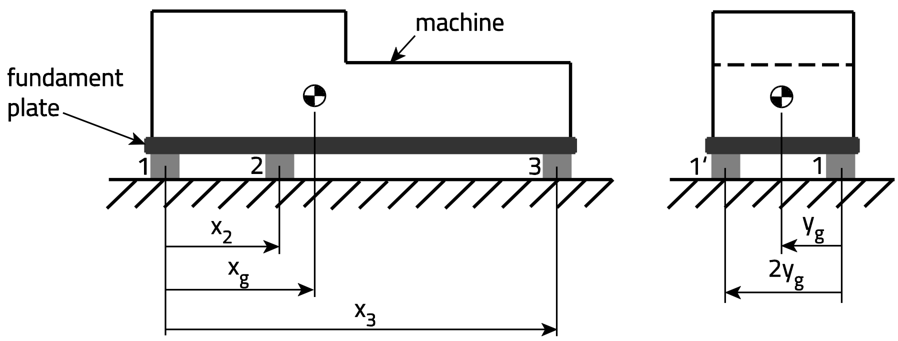

Fig. 15 – Positions of spring packets (example)